6.4b Exponential Growth and Decay $\frac{dy}{dt} = k \cdot y$ $y = y_0 e^{-kt}$

Newton's Law of Cooling: The rate at which an object's temperature is changing is directly proportional to the difference between its temperature and the temperature of the surrounding medium.

T = Temp of object

t = time

T_s = Temp of surroundings (constant)

T_0 = initial Temp of object

$y = T - T_s$

$y_0 = T_0 - T_s$

$$\frac{d(T-T_s)}{dt} = -k(T-T_s)$$

$$T - T_s = (T_0 - T_s)e^{-kt}$$

$$T = T_s + (T_0 - T_s)e^{-kt}$$

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A hard boiled egg at 98 degrees Celsius is put in a pan under running 18 degree water to cool. After 5 minutes, the egg's temperature is found to be 38 degrees. How much longer will it take the egg to reach 20 degrees?

$$38 = 18 + (98-18)e^{-k \cdot 5}$$

$$20 = 80e^{-5k}$$

$$\frac{20}{80} = e^{-5k}$$

$$\ln \frac{1}{4} = -5k$$

$$k = \frac{\ln \frac{1}{4}}{-5} = .2773$$

$$20 = 18 + 80e^{-.2773t}$$

$$\frac{2}{80} = e^{-.2773t}$$

$$\ln \frac{2}{80} = -.2773t$$

$$t = \frac{\ln \frac{2}{80}}{-.2773} = 13.3$$

$$13.3 - 5 = \boxed{8.3}$$

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1.1 1.2 Newtons La...dge

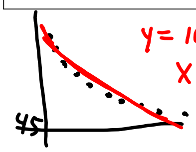
Newton's Law of Cooling

A cup of hot water was placed in the fridge. The temperature was checked at somewhat regular intervals. See the data on the next page. Make a scatter plot and find the exponential regression equation.

1.1 1.2 Newtons La...dge

	A time	B temp	C	D
1	0	124		
2	5	118		
3	10	114		
4	16	109		
5	20	106		

B temp



$y = 105(.997)^x$

X axis is HA

$y = (T_0 - T_s)e^{-kx} + T_s$

$y = 70.8(.9908)^x$ fits shifted data

$y = 70.8(.9908)^x + 45$ fits the original data

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Separation of Variables

Solve for y if $\frac{dy}{dx} = (xy)^2$ and y=1 when x=1

$$\frac{dy}{dx} = x^2y^2$$

$$\int \frac{dy}{y^2} = \int x^2\,dx$$

$$\int y^{-2}\,dy = \int x^2\,dx$$

$$\frac{y^{-1}}{-1} = \frac{x^3}{3} + c$$

$$\frac{-1}{y} = \frac{x^3}{3} + c$$

$$\frac{-1}{1} = \frac{1}{3} + c$$

$$-\frac{4}{3} = c$$

$$-\frac{1}{y} = \frac{x^3}{3} - \frac{4}{3} = \frac{x^3-4}{3}$$

$$y = \frac{-3}{x^3-4} = \frac{3}{4-x^3}$$

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